

Polarization in semileptonic $B \rightarrow X\tau$ decays*

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Received: 14 January 1999 / Revised version: 14 May 1999 / Published online: 14 October 1999

Abstract. The paper gives the polarization of the tau lepton in the semileptonic B decays with respect to the direction of the virtual W boson. The result given includes the nonperturbative HQET corrections. The perturbative QCD corrections are probably negligible, as is suggested by the existing results for the longitudinal polarization of the charged lepton (Jeżabek and Urban, 1998).

1 Introduction

The interest in semileptonic B decays is currently increasing as the B factory in KEK is scheduled to begin to collect data later this year. This domain of physics is likely to upgrade our knowledge of the standard model parameters as well as to provide tests on its validity. The semileptonic B decays can contribute to the former as their theoretical description is now far more successful than that of the hadronic processes [1–5].

The polarization of the charged lepton does not depend on the Cabibbo–Kobayashi–Maskawa matrix element, and so it can be instrumental in finding the quark masses. In a previous work, we have found the longitudinal polarization of the tauon, including first-order perturbative QCD corrections, analytically [6] by taking the analytical decay width for the unpolarized case [7] and calculating the width for a negative polarization. The result can then be integrated to give polarized tau energy distribution. The method used in that calculation can easily be modified to give other polarizations. This fact matters experimentally inasmuch as it is the polarization along the mediating W-boson direction that is easier to measure [8], see also [9]. The reason is that the direction of the τ lepton can be determined at B factories with rather poor accuracy. On the other hand, the direction of W is opposite to the direction of hadrons in semileptonic B decays. The latter can be well measured at least for the exclusive $B \rightarrow D\tau\bar{\nu}_\tau$ and $B \rightarrow D^*\tau\bar{\nu}_\tau$ channels, which probably contribute the dominant contribution to the inclusive decay rate.

The present calculation includes tree-level and HQET corrections only. We are unable to calculate perturbative QCD corrections because the analytic structure of expressions is far more complicated than in the case of the longitudinal polarization [6]. However, indications exist that the effects of perturbative QCD corrections on τ polar-

ization are negligible. In particular, the above-mentioned longitudinal polarization does not change visibly after the first-order perturbative corrections have been included either in the rest frame of the W boson [10] or that of the decaying quark [6].

The paper is broken up into four sections. In Sect. 2, kinematical variables are introduced. Sections 3 and 4 explain the method used in the calculation, and then the results are shown in Sect. 5. In Appendix A, some details of HQET calculations are explained including the discussion of singularity problems. Such problems have also been encountered in [11], where a method was proposed for eliminating them.

2 Kinematical variables

In this section, we define the kinematical variables used throughout the article as well as their boundaries. The calculation is performed in the rest frame of the decaying B meson, which coincides with that of the b quark at the tree level in the parton model. The four-momenta of the particles are denoted as follows: Q for the b quark, q for the c quark, W for the virtual W boson, τ for the charged lepton, and ν for the corresponding antineutrino. All the particles are assumed to be on shell so that their squared four-momenta equal their masses:

$$Q^2 = m_b^2, \quad q^2 = m_c^2, \quad \tau^2 = m_\tau^2, \quad \nu^2 = 0. \quad (1)$$

The employed variables are scaled to the units of the decaying quark mass m_b :

$$\rho = \frac{m_c^2}{m_b^2}, \quad \eta = \frac{m_\tau^2}{m_b^2}, \quad y = \frac{2E_\tau}{m_b}, \quad t = \frac{W^2}{m_b^2}, \quad x = \frac{2E_\nu}{m_b}. \quad (2)$$

Henceforth we scale all quantities so that $m_b^2 = Q^2 = 1$. The charged lepton is described by the light-cone variables:

$$\tau_\pm = \frac{1}{2} \left(y \pm \sqrt{y^2 - 4\eta} \right). \quad (3)$$

* Work supported in part by KBN grant 2P03B08414 and by BMBF grant POL-239-96

The W boson is characterized likewise:

$$w_0 = \frac{1}{2}(1 + t - \rho), \quad (4)$$

$$w_3 = \sqrt{w_0^2 - t}, \quad (5)$$

$$w_{\pm} = w_0 \pm w_3. \quad (6)$$

The phase space is defined by the ranges of the kinematical variables:

$$2\sqrt{\eta} \leq y \leq 1 + \eta - \rho = y_m, \quad (7)$$

$$t_{\min} = \tau_- \left(1 - \frac{\rho}{1 - \tau_-}\right) \leq t \leq \tau_+ \left(1 - \frac{\rho}{1 - \tau_+}\right) \\ = t_{\max}. \quad (8)$$

The limits above are obtained within the parton model approximation. They change if we allow for Fermi motion, which we must in order to be able to discuss the HQET corrections to the decay widths [11–15]. Also, contrary to the parton model case, the energy of neutrino can vary within limits which depend in a nontrivial manner on the values of the variables y, t . The details of this subject have been discussed in [12], so we will state here only that the integrations involving delta functions and their derivatives have the effect of confining the range of the variables y, t to that of the parton model.

3 Polarization evaluation

The polarization is found by evaluation of the unpolarized decay width and any of the two corresponding to a definite polarization, according to the definition,

$$P = \frac{\Gamma^+ - \Gamma^-}{\Gamma^+ + \Gamma^-} = 1 - 2\frac{\Gamma^-}{\Gamma}, \quad (9)$$

where $\Gamma = \Gamma^+ + \Gamma^-$. The calculation of the polarized width is structured in the manner of that which has yielded the longitudinal polarization [6]. Thus in the rest frame of the decaying quark, one can decompose:

$$s = \mathcal{A}Q + \mathcal{B}W. \quad (10)$$

The coefficients \mathcal{A}, \mathcal{B} can be evaluated with the conditions defining the polarization four-vector s ; these conditions reduce to the following when the parton model value of the neutrino energy is assumed:

$$\mathcal{A}_0^{\pm} = \mp \frac{t + \eta}{\sqrt{t(y - y_-)(y_+ - y)}}, \quad (11)$$

$$\mathcal{B}_0^{\pm} = \pm \frac{y}{\sqrt{t(y - y_-)(y_+ - y)}}, \quad (12)$$

where the superscripts at \mathcal{A}, \mathcal{B} denote the polarization of the lepton, while

$$y_{\pm} = (1 + \eta/t)w_{\pm}. \quad (13)$$

This observation is made relevant by the fact that the decay width for a definite polarization of the charged lepton is gotten from the analogous expression for the unpolarized case,

$$d\Gamma_0 = G_F^2 M_b^5 |V_{CKM}|^2 \mathcal{M}_{0,3}^{\text{un}} d\mathcal{R}_3(Q; q, \tau, \nu) / \pi^5 \quad (14)$$

where

$$\mathcal{M}_{0,3}^{\text{un}}(\tau) = q \cdot \tau Q \cdot \nu, \quad (15)$$

by formal replacement of the lepton four-momentum by the following four-vector K :

$$K = \tau - m_{\tau} s. \quad (16)$$

Then we obtain

$$\mathcal{M}_{0,3}^{\text{pol}} = \frac{1}{2} \mathcal{M}_{0,3}^{\text{un}}(K = \tau - ms) = \frac{1}{2} (q \cdot K)(Q \cdot \nu). \quad (17)$$

Although the expressions above are written for the Born approximation, corrections received by the hadronic tensor obviously do not alter this scheme, so we can apply it to the HQET calculations, too. However, we need to reevaluate the coefficients in the decomposition (10), taking into account the Fermi motion and working in the rest frame of the decaying meson. For a derivation of these, cf. Appendix A. Applying now the representation (10) of the polarization four-vector s we readily obtain the following useful formula for the matrix element with the lepton polarized:

$$\mathcal{M}^{\pm} = \frac{1}{2} \mathcal{M}^{\text{un}}(\tau) \pm \frac{\sqrt{\eta}}{\sqrt{y(t + \eta)(x + y) - y^2 t - (t + \eta)^2}} \\ \times [y \mathcal{M}^{\text{un}}(W) - (t + \eta) \mathcal{M}^{\text{un}}(Q)]. \quad (18)$$

The above expression is valid for the HQET corrections too. The first term on the right hand side of (18) can be calculated immediately once we know the result for the unpolarized case. Then the other terms require the formal replacement of the four-momenta, $\tau \rightarrow W$ and $\tau \rightarrow Q$, in the argument.

4 Evaluation of HQET corrections

Using the operator expansion technique, one can obtain corrections to the decay widths of heavy hadrons which effectively lead to new terms in the hadronic tensor appearing in the triple differential decay width,

$$\frac{d\Gamma}{dx dy} = \frac{|V_{cb}|^2 G_F^2}{2\pi^3} \mathcal{L}_{\mu\nu} \mathcal{W}_{\mu\nu}. \quad (19)$$

The hadronic tensor \mathcal{W} , related to an inclusive decay of a beautiful hadron H_b ,

$$\mathcal{W}_{\mu\nu} = (2\pi)^3 \sum_X \delta^4(p_{H_b} - q - p_X) \\ \times \langle H_b(v, s) | J_{\mu}^{c\dagger} | X \rangle \langle X | J_{\nu}^c | H_b(v, s) \rangle \quad (20)$$

can be expanded in the form

$$\begin{aligned} \mathcal{W}_{\mu\nu} = & -g_{\mu\nu}W_1 + v_\mu v_\nu W_2 - i\epsilon_{\mu\nu\alpha\beta}v^\alpha q^\beta W_3 \\ & + q_\mu q_\nu W_4 + (q_\mu v_\nu + v_\mu q_\nu)W_5 . \end{aligned} \quad (21)$$

The form factors W_n can be determined by use of the relation between the tensor \mathcal{W} and the matrix element of the transition operator

$$T_{\mu\nu} = -i \int d^4x e^{-iqx} T[J_\mu^\dagger(x) J_\nu^c(0)] , \quad (22)$$

which is

$$\mathcal{W}_{\mu\nu} = -\frac{1}{\pi} \text{Im} \langle H_b | T_{\mu\nu} | H_b \rangle . \quad (23)$$

The coefficients W_n of (21) have all been found elsewhere, see, e.g., [12] for a complete list. Then the distribution (19) can be schematically cast in the following form:

$$\frac{d\Gamma}{dx dt dy} = f_1 \delta(x-x_0) + f_2 \delta'(x-x_0) + f_3 \delta''(x-x_0) , \quad (24)$$

where

$$x_0 = 1 + t - \rho - y \quad (25)$$

is the value of the neutrino energy in the parton model kinematics. The triple differential distribution must be integrated over the neutrino energy to give meaningful results. The final lepton energy distribution obtained on two subsequent integrations may be trusted, except for the endpoint region where the operator product expansion fails. In the present paper, we give the double differential distribution so that the lepton energy distribution has to be obtained numerically. The calculation does not show any features that are not familiar from the cases of the other known polarizations.

5 Results

5.1 Double differential distribution

The polarized distribution can be written in the form,

$$\frac{1}{\Gamma_0} \frac{d\Gamma^\pm}{dy dt} = \frac{1}{2} F^{\text{un}} \pm \left(\tilde{F} + \tilde{F}_+ - \tilde{F}_- \right) , \quad (26)$$

where

$$\Gamma_0 = \frac{G_F^2 m_b^5}{192\pi^3} |V_{\text{CKM}}|^2 . \quad (27)$$

The first term on the right-hand side of (26) stands for the unpolarized distribution, given, e.g., in [12], (30). Here we will give only the new other term¹:

$$\begin{aligned} \tilde{F} = & \sqrt{\eta} \mathcal{W} \left[6f_1 + K_b \mathcal{W} (f_2 + f_3 \mathcal{W}^2 + \frac{3}{2} f_4 \mathcal{W}^4) \right. \\ & \left. + G_b (f_5 + f_6 \mathcal{W}^2) \right] , \end{aligned} \quad (28)$$

where

$$\begin{aligned} f_1 = & -yt(1 + \rho - \eta) + 2yt^2 - y(1 + \rho\eta - 2\rho + \rho^2 + \eta) \\ & - y^2 t + y^2(1 - \rho) + t(1 + 2\rho\eta - \rho^2) + t^2(2\rho - \eta) \\ & - t^3 - \rho^2\eta + \eta , \end{aligned} \quad (29)$$

$$\begin{aligned} f_2 = & -8yt(1 - \rho + \eta) - 16yt^2 + 8y(1 + \rho\eta - 2\rho + \rho^2 - \eta) \\ & + 6y^2 t - 6y^2(1 - \rho) + 2t(1 - 6\rho\eta - 4\rho + 3\rho^2 + 12\eta) \\ & + 2t^2(8 - 6\rho + 3\eta) + 6t^3 - 8\rho\eta + 6\rho^2\eta \\ & + 2\eta + 8\eta^2 , \end{aligned} \quad (30)$$

$$\begin{aligned} f_3 = & yt(-10\rho\eta + 6\rho\eta^2 + 14\rho^2\eta + 9\rho^2\eta^2 - 6\rho^3\eta + 2\eta \\ & + 9\eta^2 - 4\eta^3) + yt^2(1 - 9\rho\eta^2 - 5\rho + 18\rho^2\eta + 7\rho^2 \\ & - 3\rho^3 + 6\eta - 13\eta^2) + yt^3(1 - 18\rho\eta - 2\rho + 9\rho^2 - 14\eta \\ & + 3\eta^2) - yt^4(5 + 9\rho - 6\eta) + 3yt^5 + y(-5\rho\eta^2 + 4\rho\eta^3 \\ & + 7\rho^2\eta^2 - 3\rho^3\eta^2 + \eta^2 + 4\eta^3) + y^2 t(1 + 14\rho\eta + 2\rho\eta^2 \\ & - 3\rho - \rho^2\eta + 3\rho^2 - \rho^3 - 13\eta + 8\eta^2) + y^2 t^2(-6 + 5\rho\eta \\ & + 6\rho + 15\eta - \eta^2) + y^2 t^3(7 + 3\rho - 3\eta) - 2y^2 t^4 + y^2\eta \\ & \times (-3\rho + 8\rho\eta + 3\rho^2 - \rho^2\eta - \rho^3 - 7\eta + 1) \\ & + y^3 t(1 - \rho)^2 - y^3 t^2\eta - y^3 t^3 + y^3\eta(1 - \rho)^2 , \end{aligned} \quad (31)$$

$$\begin{aligned} f_4 = & y^2 t(-6\rho\eta^2 - 2\rho^2\eta^3 + 6\rho^3\eta^2 + 4\rho^3\eta^3 - 3\rho^4\eta^2 + 3\eta^2 \\ & - 2\eta^3) + y^2 t^2(-6\rho\eta - 2\rho\eta^3 - 6\rho^2\eta^2 - 6\rho^2\eta^3 + 6\rho^3\eta \\ & + 12\rho^3\eta^2 - 3\rho^4\eta + 3\eta - 6\eta^2) + y^2 t^3(1 - 6\rho\eta^2 + 4\rho\eta^3 \\ & - 2\rho - 6\rho^2\eta - 18\rho^2\eta^2 + 12\rho^3\eta + 2\rho^3 - \rho^4 - 6\eta \\ & + 2\eta^3) + y^2 t^4(-2 - 6\rho\eta + 12\rho\eta^2 - 18\rho^2\eta - 2\rho^2 \\ & + 4\rho^3 + 6\eta^2 - \eta^3) + y^2 t^5(12\rho\eta - 2\rho - 6\rho^2 + 6\eta \\ & - 3\eta^2) + y^2 t^6(2 + 4\rho - 3\eta) - y^2 t^7 + y^2(-2\rho\eta^3 \\ & + 2\rho^3\eta^3 - \rho^4\eta^3 + \eta^3) + y^3 t(8\rho\eta + \rho\eta^2 + 2\rho\eta^3 \\ & - 12\rho^2\eta + \rho^2\eta^2 + 3\rho^2\eta^3 + 8\rho^3\eta - \rho^3\eta^2 - 2\rho^4\eta - 2\eta \\ & - \eta^2 + 3\eta^3) + y^3 t^2(-1 - \rho\eta + 6\rho\eta^2 - 3\rho\eta^3 + 4\rho \\ & - \rho^2\eta + 9\rho^2\eta^2 - 6\rho^2 + \rho^3\eta + 4\rho^3 - \rho^4 + \eta + 9\eta^2 \\ & - 3\eta^3) + y^3 t^3(1 + 6\rho\eta - 11\rho\eta^2 - \rho + 9\rho^2\eta - \rho^2 + \rho^3 \\ & + 9\eta - 11\eta^2 + \eta^3) + y^3 t^4(3 - 13\rho\eta + 2\rho + 3\rho^2 - 13\eta \\ & + 4\eta^2) + y^3 t^5(-5 - 5\rho + 5\eta) + 2y^3 t^6 + y^3(4\rho\eta^2 \\ & + \rho\eta^3 - 6\rho^2\eta^2 + \rho^2\eta^3 + 4\rho^3\eta^2 - \rho^3\eta^3 - \rho^4\eta^2 - \eta^2 \\ & - \eta^3) + y^4 t(-6\rho\eta + 2\rho\eta^2 + 6\rho^2\eta + \rho^2\eta^2 - 2\rho^3\eta + 2\eta \\ & - 3\eta^2) + y^4 t^2(1 + 4\rho\eta + \rho\eta^2 - 3\rho + 2\rho^2\eta + 3\rho^2 - \rho^3 \\ & - 6\eta + 3\eta^2) + y^4 t^3(-3 + 2\rho\eta + 2\rho + \rho^2 + 6\eta - \eta^2) \\ & + y^4 t^4(3 + \rho - 2\eta) - y^4 t^5 + y^4(-3\rho\eta^2 + 3\rho^2\eta^2 \\ & - \rho^3\eta^2 + \eta^2) , \end{aligned} \quad (32)$$

$$\begin{aligned} f_5 = & 4yt(3 + 5\rho - 5\eta) - 40yt^2 + 4y(1 + 5\rho\eta - 6\rho + 5\rho^2 \\ & + \eta) + 10y^2 t - 2y^2(1 - 5\rho) - 6t(1 + 10\rho\eta - 5\rho^2) \\ & - 2t^2(4 + 30\rho - 15\eta) + 30t^3 \\ & + 30\rho^2\eta - 6\eta + 8\eta^2 , \end{aligned} \quad (33)$$

$$\begin{aligned} f_6 = & yt(-18\rho\eta - 2\rho\eta^2 + 6\rho^2\eta - 15\rho^2\eta^2 + 10\rho^3\eta + 2\eta \\ & + 17\eta^2 - 4\eta^3) + yt^2(1 - 16\rho\eta + 15\rho\eta^2 - 9\rho - 30\rho^2\eta \\ & + 3\rho^2 + 5\rho^3 + 10\eta - \eta^2) + yt^3(1 + 30\rho\eta - 10\rho \\ & - 15\rho^2 + 10\eta - 5\eta^2) + yt^4(7 + 15\rho - 10\eta) - 5yt^5 \end{aligned}$$

¹ A FORTRAN code for this formula is available from piotr@charm.phys.us.edu.pl

$$\begin{aligned}
& +y(-9\rho\eta^2 + 4\rho\eta^3 + 3\rho^2\eta^2 + 5\rho^3\eta^2 + \eta^2 + 8\eta^3) \\
& +y^2t(-1 + 30\rho\eta - 10\rho\eta^2 + 7\rho + 5\rho^2\eta - 11\rho^2 + 5\rho^3 \\
& - 11\eta - 4\eta^2) + y^2t^2(-2 - 25\rho\eta + 18\rho - 19\eta + 5\eta^2) \\
& - 15y^2t^3(1 + \rho - \eta) + 10y^2t^4 + y^2(7\rho\eta + 12\rho\eta^2 \\
& - 11\rho^2\eta + 5\rho^2\eta^2 + 5\rho^3\eta - \eta - 9\eta^2) \\
& + y^3t(1 - 6\rho + 5\rho^2 + 8\eta) + y^3t^2(8 - 5\eta) - 5y^3t^3 \\
& + y^3(-6\rho\eta + 5\rho^2\eta + \eta) , \tag{34}
\end{aligned}$$

$$\begin{aligned}
\tilde{F}_{\pm} = & \sqrt{\eta} \mathcal{W}_{\pm} \{ [K_b (h_{1,\pm} + h_{2,\pm} \mathcal{W}_{\pm}^2) + G_b h_{3,\pm}] \delta(z_{\pm}) \\
& + K_b h_{4,\pm} \delta'(z_{\pm}) \} , \tag{35}
\end{aligned}$$

where

$$\begin{aligned}
h_{1,\pm} = & -8yt\eta + 8y\eta^2 + 4y^2t + 12y^2\eta + 2y^3t - 2y^3\eta \\
& - 2y^4 - 16t\eta - 16\eta^2 - 4\sigma_{\pm} (6yt - 2y\eta + 3y^2t \\
& + y^2\eta - y^3 - 8t\eta - 4t^2 - 4\eta^2) - 8\sigma_{\pm}^2 (3yt - y\eta \\
& - 5y^2 + 4t + 4\eta) + 16\sigma_{\pm}^3 (3y + t + \eta) , \tag{36}
\end{aligned}$$

$$\begin{aligned}
h_{2,\pm} = & 2\sigma_{\pm} y (8t\eta^2 + 16t^2\eta + 8t^3 - 4yt\eta^2 + 4yt^3 - 8y^2t\eta \\
& - 6y^2t^2 - 2y^2\eta^2 - y^3t^2 + y^3\eta^2 + y^4t + y^4\eta) \\
& + 4\sigma_{\pm}^2 y (-4t\eta^2 - 8t^2\eta - 4t^3 - 12yt\eta - 8yt^2 \\
& - 4y\eta^2 + 2y^2t\eta - y^2t^2 + 3y^2\eta^2 + 3y^3t + 3y^3\eta) \\
& + 8\sigma_{\pm}^3 y (-4t\eta - 2t^2 - 2\eta^2 + 4yt\eta + yt^2 + 3y\eta^2 \\
& + 3y^2t + 3y^2\eta) + 16\sigma_{\pm}^4 y (2t\eta + t^2 + \eta^2 \\
& + yt + y\eta) , \tag{37}
\end{aligned}$$

$$\begin{aligned}
h_{3,\pm} = & -16yt\eta - 8yt + 8y^2t + 8y\eta + 8y\eta^2 - 8y^2t + 8y^2\eta \\
& + 24t^2 - 24\eta^2 + 4\sigma_{\pm} (-10yt - 14y\eta - 5y^2t \\
& + 5y^2\eta - 4y^2 + 5y^3 + 12t - 4t^2 + 12\eta + 4\eta^2) \\
& + 16\sigma_{\pm}^2 (5y\eta - 2y + 5y^2 - 9t - 9\eta) \\
& + 80\sigma_{\pm}^3 (y + t + \eta) , \tag{38}
\end{aligned}$$

$$\begin{aligned}
h_{4,\pm} = & 4\sigma_{\pm} (4yt\eta - 4yt^2 + 6y^2t + 2y^2\eta + y^3t - y^3\eta \\
& - y^4 - 8t\eta - 8t^2) + 8\sigma_{\pm}^2 (8yt + 4y\eta + y^2t - 3y^2\eta \\
& - 3y^3 + 4t\eta + 4t^2) - 16\sigma_{\pm}^3 (yt + 3y\eta + 3y^2 \\
& - 2t - 2\eta) - 32\sigma_{\pm}^4 (y + t + \eta) , \tag{39}
\end{aligned}$$

and

$$\mathcal{W}_{\pm} = \frac{1}{\sqrt{y(t+\eta)(2\sigma_{\pm} + y) - y^2t - (t+\eta)^2}} , \tag{40}$$

$$\mathcal{W} = \frac{1}{\sqrt{y(t+\eta)(x_0 + y) - y^2t - (t+\eta)^2}} , \tag{41}$$

$$\sigma_{\pm} = (t - \eta)/(2\tau_{\pm}) , \quad z_{\pm} = 1 + t - \rho - y - 2\sigma_{\pm} . \tag{42}$$

The parameters K_b, G_b , representing the kinetic energy and the chromomagnetic energy, are defined according to [13].

5.2 Lepton energy distribution

As regards the HQET correction terms, we give the energy distribution only in the form of a diagram evaluated numerically. Below, we also give the Born-level approximation analytically. The analytic formulas for the polarized distribution can be simplified if we split the kinematical range of y into two parts, separated by the value of the charged lepton energy where the virtual W boson can stay at rest. This value is

$$y_W = 1 - \sqrt{\rho} + \frac{\eta}{1 - \sqrt{\rho}} . \tag{43}$$

In the formulas below, the superscripts A, B refer to the appropriate regimes:

$$y < y_W \quad \text{region } A , \tag{44}$$

$$y > y_W \quad \text{region } B . \tag{45}$$

The energy distribution of the polarized τ lepton reads

$$\frac{d\Gamma^{\pm}}{dy} = 12\Gamma_0 \left[\frac{1}{2} f(y) \pm \Delta f(y) \right] . \tag{46}$$

The function $f(y)$ represents the unpolarized case,

$$\begin{aligned}
f(y) = & \frac{1}{6} \zeta^2 \sqrt{y^2 - 4\eta} \{ \zeta [y^2 - 3y(1 + \eta) + 8\eta] \\
& + (3y - 6\eta)(2 - y) \} , \tag{47}
\end{aligned}$$

with

$$\zeta = 1 - \frac{\rho}{1 + \eta - y} . \tag{48}$$

The function $\Delta f(y)$ reads

$$\Delta f(y) = \frac{3}{8} \sqrt{\eta|y - 1|} \phi_1 \Psi + \frac{1}{4} \eta \phi_2 , \tag{49}$$

with

$$\begin{aligned}
\phi_1 = & -5\lambda^3/(y - 1)^4 + 3\lambda(4\eta - \lambda - \lambda^2)/(y - 1)^3 + (4\eta\lambda \\
& - 4\eta + \lambda + 7\lambda^2 + \lambda^3)/(y - 1)^2 + (-1 + 4\eta\lambda - 28\eta \\
& + 15\lambda - \lambda^2 - \lambda^3)/(y - 1) - 1 + 12y\eta - 11y\lambda \\
& + 7y - y^2 + 12\eta\lambda - 24\eta + 14\lambda - 11\lambda^2 , \tag{50}
\end{aligned}$$

$$\begin{aligned}
\phi_2^A = & \sqrt{y^2 - 4\eta} [15\lambda^2\xi/(y - 1)^3 + (10\eta\lambda\xi^2 - 16\eta\xi \\
& + 24\lambda\xi - 10\lambda\xi^2 - 20\lambda - 6\lambda^2\xi)/(y - 1)^2 + (-4 \\
& - 14\eta\lambda\xi^2 - 48\eta\xi + 66\eta\xi^2 - 24\eta\xi^3 + 8\eta^2\xi^3 - 76\lambda\xi \\
& + 14\lambda\xi^2 + 48\lambda + 3\lambda^2\xi + 25\xi - 26\xi^2 + 8\xi^3)/(y - 1) \\
& + 3 - 3y + 57\eta\xi - 22\eta\xi^2 - 12\lambda\xi - 21\lambda + 34\xi \\
& - 18\xi^2 + 8\xi^3] , \tag{51}
\end{aligned}$$

$$\begin{aligned}
\phi_2^B = & 15\zeta\lambda^2/(y - 1)^3 + (60\eta\zeta\lambda - 16\eta\zeta - 30\eta\zeta^2\lambda - 16\zeta\lambda \\
& - 21\zeta\lambda^2 + 10\zeta^2\lambda)/(y - 1)^2 + (-104\eta\zeta\lambda - 84\eta\zeta \\
& + 52\eta\zeta^2\lambda + 122\eta\zeta^2 - 40\eta\zeta^3 + 160\eta^2\zeta - 160\eta^2\zeta^2 \\
& + 40\eta^2\zeta^3 - 24\zeta\lambda + 9\zeta\lambda^2 + 17\zeta - 4\zeta^2\lambda - 22\zeta^2
\end{aligned}$$

$$\begin{aligned}
& +8\zeta^3)/(y-1) + 18 - 29y\eta + 27y\lambda - 21y + 3y^2 \\
& +46\eta\zeta\lambda - 59\eta\zeta - 14\eta\zeta^2\lambda + 78\eta\zeta^2 - 16\eta\zeta^3 - 71\eta\lambda \\
& +59\eta - 43\eta^2\zeta + 26\eta^2\zeta^2 - 8\eta^2\zeta^3 + 46\eta^2 - 6\zeta\lambda \\
& -3\zeta\lambda^2 + 69\zeta - 6\zeta^2\lambda - 52\zeta^2 + 16\zeta^3 \\
& -42\lambda + 24\lambda^2, \tag{52}
\end{aligned}$$

where

$$\xi = 2 - \zeta, \quad \lambda = \rho + \eta. \tag{53}$$

The function Ψ can be written in the form

$$\Psi = \begin{cases} \arccos \omega_{\min} - \arccos \omega_{\max}, & y < 1 \\ \operatorname{arcosh} \omega_{\max} - \operatorname{arcosh} \omega_{\min}, & y > 1 \end{cases} \tag{54}$$

with

$$\omega_{\min, \max} = \frac{2(y-1)t_{\min, \max} + y(y_m - y) - 2\eta}{y\sqrt{(y_m - y)^2 + 4\eta\rho}}. \tag{55}$$

Because of terms containing inverse powers of $(y-1)$, the expression (49) for $\Delta f(y)$ is apparently divergent at $y=1$. However, expanding $\Delta f(y)$ in powers of $(y-1)$, for $y < 1$ and $y > 1$ one can check that this function is regular at $y=1$.

The HQET contribution to the decay distributions is known to render them unreliable near the endpoint values of the tauon energy. This ambiguity reveals itself in the polarization as well. Similar problems appear in calculations of perturbative corrections [16,17]. All these problems are cured if instead of distributions, their moments are considered [10,18,19]. In the case of τ polarization, a better-defined quantity is the integrated polarization

$$\begin{aligned}
P_{\text{int}}(\bar{y}) &= \int_{y_{\min}}^{\bar{y}} dy \left(\frac{d\Gamma^+}{dy} - \frac{d\Gamma^-}{dy} \right) \\
& \quad / \int_{y_{\min}}^{\bar{y}} dy \left(\frac{d\Gamma^+}{dy} + \frac{d\Gamma^-}{dy} \right) \tag{56}
\end{aligned}$$

where both the lowest-order perturbative and the HQET terms are included. In Fig. 1, the integrated polarization is shown as a function of the scaled energy y of the τ lepton. The lowest-order prediction corresponds to the dashed line, and the solid line is obtained with the inclusion of HQET corrections. The question that arises is whether the perturbative QCD corrections can change this result significantly. As has already been suggested in the introduction, it is plausible that no such thing happens.

On integration over the whole range of the charged lepton energy, one arrives at the total polarization at the tree level corrected for the $O(1/m_b^2)$ effects as predicted by HQET. For $m_b = 4.75 \text{ GeV}$ and $m_c = 1.35 \text{ GeV}$, we obtain

$$P = -0.7235 + 4.21 \frac{K_b}{m_b^2} + 1.48 \frac{G_b}{m_b^2}. \tag{57}$$

Taking $K_b = 0.15 \text{ GeV}^2$ and $G_b = -0.18 \text{ GeV}^2$, we obtain $P = -0.706$.

Although we are mostly concerned with the tau lepton polarization here, the formulas derived in the present

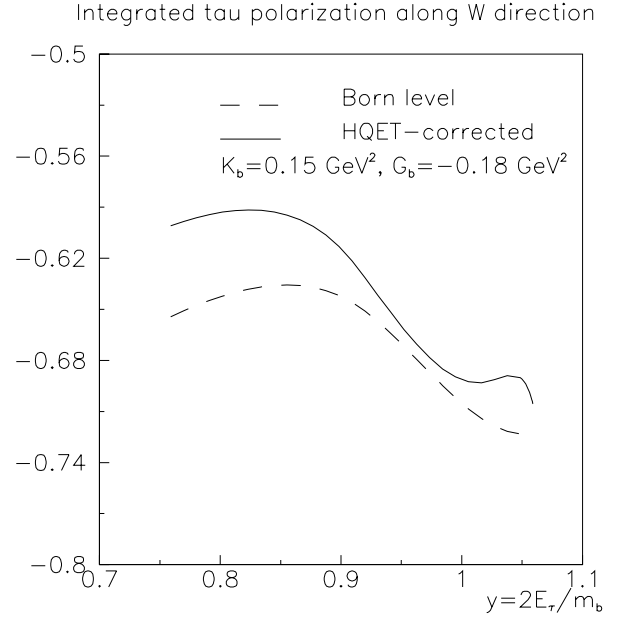


Fig. 1. Integrated polarization of the τ lepton along the direction of virtual W in the Born approximation (dashed) and including HQET corrections (solid) as functions of the scaled τ energy y . The mass of the b quark is taken to be 4.75 GeV , and the c quark, 1.35 GeV

work may well be used in evaluating the polarization of the light leptons. Interestingly, in the limit of a vanishing mass of the charged lepton the polarization falls to zero, apart from the endpoints; cf. (49) and (28). It is due to the chiral $V - A$ structure of the weak charged current that, according to (17), the decay widths with a definite polarization differ by a term proportional to $m_\tau s^\mu$. The polarization four-vector of the charged lepton can be decomposed as follows:

$$s^\mu = (s^0, \vec{s}) = \left(\frac{p}{m} \sqrt{1 - (\vec{s}_\perp)^2}, \vec{s}_\perp, \frac{E}{m} \sqrt{1 - (\vec{s}_\perp)^2} \right), \tag{58}$$

where \vec{s}_\perp is understood to mean the part of the three-vector \vec{s} perpendicular to the direction of the charged lepton. The quantities E and p denote, respectively, the energy and the three-momentum value of the charged lepton. This form can easily be seen to meet the definition of the polarization four-vector; cf. Appendix A. As the lepton mass approaches zero, (58) gives

$$ms^\mu \approx \sqrt{1 - (\vec{s}_\perp)^2} \tau^\mu + m (0, \vec{s}_\perp, 0). \tag{59}$$

However, if we want to keep the angle subtended by the polarization vector and the lepton momentum constant, the parallel part of the polarization should be proportional to the perpendicular one, thereby forcing the factor of $\sqrt{1 - (\vec{s}_\perp)^2}$ to be of order m/E . Then the right-hand side of (59) tends to zero for $m \rightarrow 0$. For the vanishing charged lepton mass the polarization can be nonzero only where the virtual W boson is collinear with the charged lepton. In general, the contribution to polarization is ap-

preciable for only the W direction within the cone defined by the condition

$$|\vec{s}_\perp|/|\vec{s}| = \mathcal{O}(m/E). \quad (60)$$

In particular, this happens if p is much larger than the energy of the neutrino. For semitaonic B decays, the condition (60) is satisfied in the whole phase space and the resulting polarization is fairly large.

6 Summary

The polarization of the tau lepton along the W -boson direction in semileptonic B decays has been found at the tree level in perturbative QCD, and the leading-order HQET corrections have been included. The quantity is of experimental interest. The fact that it does but slowly vary of the charged lepton is rather favorable in this context [8]. The QCD one-loop corrections are unknown but their irrelevance for the longitudinal polarization both in the rest frame of the decaying quark and that of the W boson indicates that no great change is to be expected once they are incorporated.

Acknowledgements. We would like to express our gratefulness to Professor M. Różańska whose suggestions have provided the experimental rationale for this work.

A The HQET calculations

We will presently construct the four-vector s representing a charged lepton polarized along the direction of the W boson. The defining properties of s are

$$s^2 = -1 \quad (61)$$

and

$$s \cdot \tau = 0, \quad (62)$$

complemented by the relation $\vec{s} \parallel \vec{W}$. Since we are working in the rest frame of the decaying meson, the four-vector s can be decomposed as

$$s = \mathcal{A}v + \mathcal{B}W, \quad (63)$$

where v and W stand for the four-velocity of the B meson and the four-momentum of the intermediate W boson, respectively. While this form automatically satisfies $\vec{s} \parallel \vec{W}$, the other two relations (61) and (62) have to be imposed, hence yielding the expressions for the coefficients appearing in (63). With $v = (1, 0, 0, 0)$, one readily identifies:

$$v \cdot \tau = y/2, \quad v \cdot \nu = x/2, \quad v^2 = -1. \quad (64)$$

These combined with the other dot products lead to the following formulas:

$$\mathcal{A}_\pm = \frac{\mp(t + \eta)}{\sqrt{y(t + \eta)(y + x) - y^2t - (t + \eta)^2}}, \quad (65)$$

$$\mathcal{B}_\pm = \frac{\pm y}{\sqrt{y(t + \eta)(y + x) - y^2t - (t + \eta)^2}}. \quad (66)$$

The evaluation of the HQET corrections involves differentiation over the neutrino energy, either once or twice. The denominator in the above expressions is easily seen to vanish at the point where the W boson is at rest. It is known that the kinematics of the process, together with the delta functions and their derivatives, finally reduces to integration over the partonic phase space. Then there is one point where the denominator vanishes:

$$y = 1 - \sqrt{\rho} + \frac{\eta}{1 - \sqrt{\rho}}, \quad (67)$$

$$t = (1 - \sqrt{\rho})^2. \quad (68)$$

One might thus raise the question of analyticity of the expressions obtained in this way. However, the divergences cancel, and moreover, the resulting distribution is continuous if we ignore the endpoint behavior. That this is indeed so may be verified by changing the variables from t to the square of the three-momentum of the W boson. Then the singularity makes its presence only on integration over w_3^2 rather than affecting the analytical structure of the distributions. It turns out that using this variable, one obtains an analytic expression. This becomes clear once one notices that the only terms that occur in the course of the calculation are the dot products of the four-vector s and the other four-vectors. Writing them out explicitly,

$$s_+ \cdot v = \frac{\tau_3 \cos \theta}{\sqrt{y^2 - \tau_3 \cos^2 \theta}}, \quad (69)$$

$$s_+ \cdot W = \frac{(y + x)\tau_3 \cos \theta - 2yw_3}{2\sqrt{y^2 - \tau_3 \cos^2 \theta}}, \quad (70)$$

with

$$\cos \theta = \frac{w_3^2 - \eta - (x - y)/4}{w_3\tau_3}, \quad (71)$$

where the subscript denotes the polarization direction, we easily verify that the triple differential distribution is analytic in the neutrino energy. Lastly, let us note that another change of variable can be useful for evaluating the distribution; namely, using the cosine of the angle subtended by the tau lepton and the neutrino [11] eliminates the singular terms from the double differential distribution. We have checked numerically that the resulting distribution is the same.

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